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Hierarchical Bases for the Indefinite Helmholtz Equation

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Abstract

For the indefinite Helmholtz equation, straightforward multigrid solvers lose their efficiency and show slow convergence or even divergence for high wave numbers. An equally insufficient performance can be observed with a standard hierarchical basis approach. The reason for this is that not all of the error frequencies can be treated by standard multigrid. Especially those frequencies which are solutions of the homogenous equation can be totally invisible for the standard solvers because of their small residuals. Brandt and Livshits suggested an approach by introducing so-called ray cycles into their multigrid scheme which consider the irremovable errors as a superposition of plane waves. We adopt this approach by using a special hierarchical basis in which the piecewise linear basis functions are multiplied by wave functions with the appropriate wave length.

Key words: hierarchical finite elements, multigrid methods, Helmholtz equation

AMS(MOS) subject classifications: 35J05, 65N22, 65N30, 65N55

1 Introduction

Brandt and Livshits [1] recently introduced a wave-ray approach for constructing an efficient multigrid solver for the 2D indefinite Helmholtz equation. The basic idea of their algorithm is that, in order to remove the error components on all scales, you have to use the appropriate tool on each level. For the high-frequency components, the Gauss-Seidel relaxation is still a good smoother. The low frequencies, for which the Gauss-Seidel relaxation diverges, can be dealt with by the slow but converging Kaczmarc-relaxation [2].

However, there remain certain error components that cannot be dealt with by any relaxation method on any grid. Brandt and Livshits call them the *principal components* as they are in fact the solutions of the homogeneous Helmholtz equation — $e^{\pm ikx}$ in 1D, $e^{\pm i(\vec{k}\cdot\vec{x})}$ in higher dimensions. As a result of this, the principal error components produce arbitrary small residuals on the fine grids, so they cannot be removed by residual-based relaxation methods. On the coarse grids, they are not resolved and therefore invisible for any relaxation method. Thus, there is no standard grid on which these error components get removed efficiently.

Brandt and Livshits therefore add so-called *ray-cycles* to their multigrid scheme. Each ray cycle operates on a transformed equation and reduces the principal error in one certain direction. To remove all principal error components Brandt and Livshits use a discretization of the *principal circle* — i.e. the circle of the principal frequencies in the frequency space — and apply ray-correction cycles in all discretized wave directions.

As a result of analysis done on Toeplitz matrices, Huckle [3] found some similar results for the special case of the Helmholtz equation:

For Toeplitz matrices, the range of eigenvalues is given by a characteristic function, which can be read directly from the coefficients of the matrix. Using the characteristic function, one can identify the error frequencies which cause the near-zero-eigenvalues and, therefore, the bad condition and slow convergence. It is then possible to design preconditioners that move the spectrum of the eigenvalues away from zero.

In 1D, it can be shown that the Helmholtz equation can be transformed into two Laplace equations. Each each of them can be interpreted as a transformed wave equation representing one of the two possible wave directions. Both Laplace equations can be solved efficiently by standard multigrid solvers.

In the 2D case, the near-zero-eigenvalues approximately build a circle in the frequency space (similar to the principal circle used by Brandt and Livshits). Using a discretization of this "circle" and applying preconditioners in every discretized direction results in a matrix with bounded condition and, therefore, leads to a fast iterative solution method (using cg, for example).

The third aspect which is important for the construction of our Helmholtz solver is the explicit use of hierarchical bases in finite element methods. It is well known that using the canonical hierarchical basis for discretizing the Poisson equation leads to a diagonal stiffness matrix in 1D. For convection or convection diffusion problems, one can design hierarchical bases or generating systems which — at least in the 1D case — result in diagonal matrices or at least matrices of $O(1)$ condition [4].

We will show in this paper that for the Helmholtz equation hierarchical bases can be constructed in a similar way and with similar success. The solution procedure integrates the ideas of the ray-correction. Thus, one gets an 1D-algorithm featuring two correction cycles. Each cycle uses a FEM method with a special hierarchical basis resulting in a diagonal stiffness matrix which is trivial to solve. For the higher dimensional case, the proposed hierarchical bases form the principle for constructing efficient multilevel methods for the ray-correction cycles.

2 The Model Problem

From now on we will focus on the 1D Helmholtz equation with constant wave number k:

$$\mathcal{L}u = u'' + k^2 u = f. \tag{1}$$

Both the solution u and the right-hand side f may have complex values. We will especially examine the highly indefinite case meaning that the wavelength $\lambda = \frac{2\pi}{k}$ shall be much smaller than the size of the computational domain.

The right-hand side f is assumed to be zero outside a compact set which shall be enclosed by the computational domain. We use the radiation boundary conditions which, in 1D, are reduced to the condition that the incoming waves are fixed at the boundaries whereas the outgoing waves may vary freely depending on the solution.

We want to find an efficient solver for the solution of the discretized equation (with standard second-order discretization):

$$\frac{1}{h^2}\left(-u_{i-1} + 2u_i - u_{i+1}\right) + \widehat{k}^2 u_i = f_i, \qquad \hat{k}^2 = \frac{2}{h^2}\left(1 - \cos kh\right). \tag{2}$$

The mesh size h is chosen so that the wave length is well resolved, i.e. $h \ll \frac{2\pi}{k}$. Using the modified wave number $\widehat{k}$ in the discretized equation ensures that its solutions have the same wavelength as the continuous equation.

3 Solution scheme

Adopting the idea of Brandt and Livshits [1], we write the error as the product of a smooth function $\hat{u}$ and a principal error component of the type e^{ikx}. Then, the 1D Helmholtz equation (1) simplifies to

$$\begin{aligned} & \left(\hat{u}e^{ikx}\right)'' + k^2 \hat{u} e^{ikx} = f \\ \iff & \left(\hat{u}'' + 2ik\hat{u}' - k^2\hat{u} + k^2\hat{u}\right) e^{ikx} = f \\ \iff & \hat{u}'' + 2ik\hat{u}' = \hat{f}. \end{aligned} \tag{3}$$

This equation resembles a convection diffusion equation in a certain way. The physical interpretation is a switch to ray optics, where the propagation of the waves follows one certain direction, given by the 1D wave vector $+k$ (or $-k$, respectively).

Because rays in the two possible directions can superpose each other, one has to solve a ray-correction equation of type (3) for both wave directions (which differs from the convection-diffusion case, where only one direction is possible). Thus, we get two equations — the one above and one for e^{-ikx}.

To ensure the consistency of the iteration scheme, the solution algorithm will be based on the residual equations. By solving the transformed equations in both possible wave directions we will compute corrections which remove the principal error components. The numerical examples will show that in 1D all error components get removed. Thus no further relaxation will be necessary.

4 Correction cycles

The correction cycles are constructed from the residual equation

$$\mathcal{L}\varepsilon = r, \qquad r = f - \mathcal{L}u. \tag{4}$$

After replacing the residual r with $\sum r_j w_j(x)$ and the error ε with $\sum u_j w_j(x)$, we get the weak formulation of equation (4):

$$\int v_l(x)\ \mathcal{L}\left(\sum_j u_j w_j(x)\right) dx = \int v_l(x)\left(\sum_j r_j w_j(x)\right) dx. \tag{5}$$

To achieve the transformation of equation (3), the ansatz functions w_j are chosen as a product of a smooth envelope function $\hat{w}_j$ and the wave term $e^{\pm ikx}$:

$$w_j(x) = \begin{cases} \hat{w}_j(x) \cdot e^{ik(x-x_j)} \\ \hat{w}_j(x) \cdot e^{-ik(x-x_j)} \end{cases}$$

We construct the test functions in the same way, except — in order to eliminate the principal components — we have to change the sign in the exponential term:

$$v_l(x) = \begin{cases} \hat{v}_l(x) \cdot e^{-ik(x-x_l)} \\ \hat{v}_l(x) \cdot e^{ik(x-x_l)} \end{cases}$$

As a result of this, the stiffness matrix L — analogous to equation (3) — solely depends on the choice of the envelope functions $\hat{w}_j$ and $\hat{v}_l$:

$$L_{lj} = \int v_l \mathcal{L} w_j\, dx = \int \hat{v}_l \left(\hat{w}_j{}'' + 2ik\hat{w}_j{}'\right)\, dx = 2ik \int \hat{v}_l \hat{w}_j{}'\, dx - \int \hat{v}_l{}' \hat{w}_j{}'\, dx. \tag{6}$$

The first choice is to take the canonical hierarchical basis for the $\hat{w}_j$. To get a diagonal stiffness matrix L, we chose the envelope test functions $\hat{v}_l$ piecewise constant as shown in figure 1. Then, as it can be seen from the plots in figure 1, the integral $\int \hat{v}_l{}' \hat{w}_j{}'\, dx$ is always zero. The integral $\int \hat{v}_l \hat{w}_j{}'\, dx$ is non-zero only for $l = j$. Thus, $L = \mathrm{diag}(4ik, \ldots, 4ik)$ is in fact a diagonal matrix.

Figure 3 shows another suitable set of ansatz and test functions which has the advantage of having an intuitive physical interpretation. The envelope

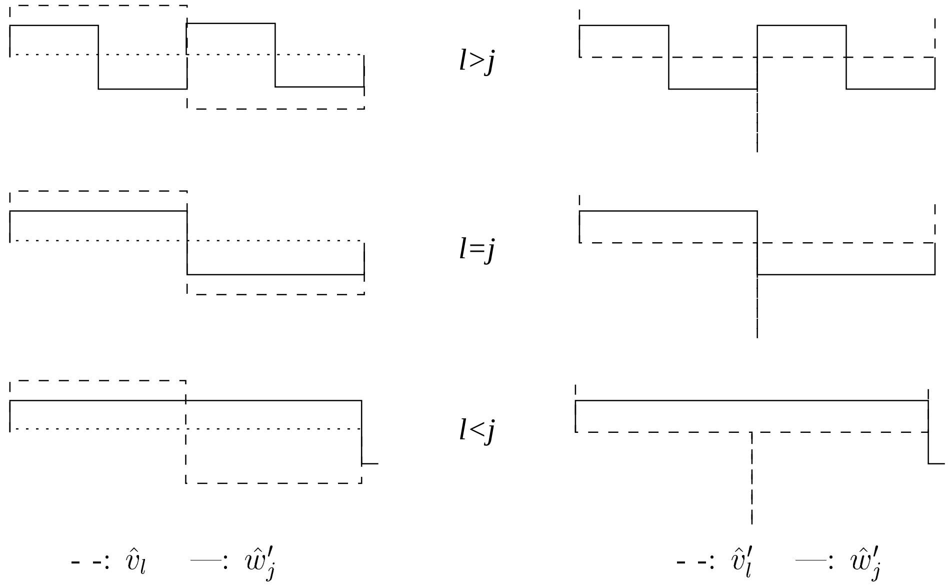


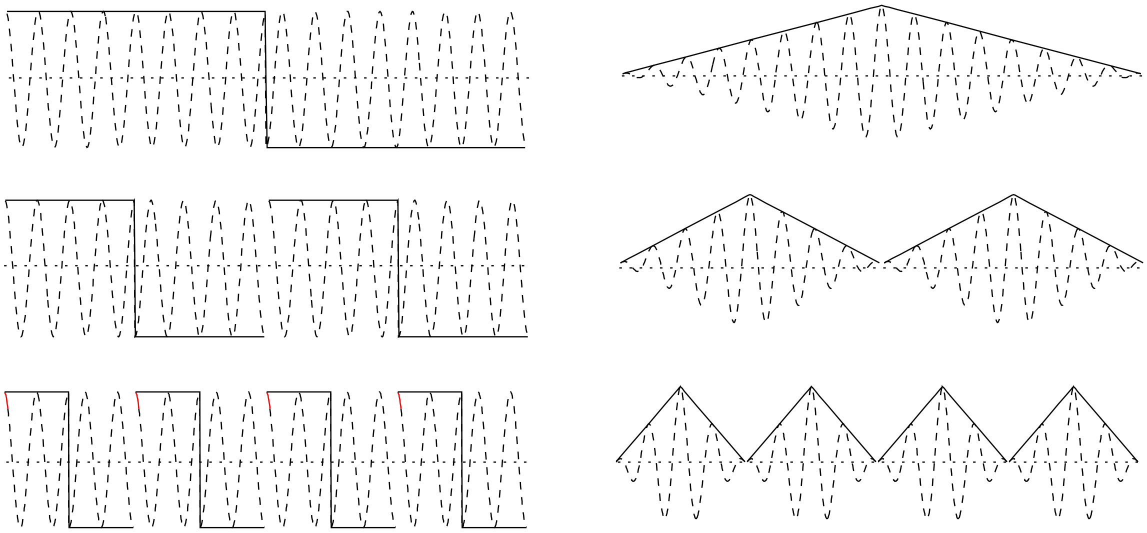
Figure 1: See equation (6): The integral $\int \hat{v}_l{}'\hat{w}_j{}'\,dx$ is always zero, the integral $\int \hat{v}_l\hat{w}_j{}'\,dx$ is non-zero only for $l = j$.

functions $\hat{w}_j$ and $\hat{v}_l$ are piecewise constant like they would be chosen for a convection-diffusion equation. To avoid Dirac functions in the first derivative we add a ramp at the "edges".

Again, one can see from equation (6) and figure 4 that the stiffness matrix L becomes diagonal:

$$L_{lj} = 2ik \int \hat{v}_l\hat{w}_j{}'\,dx - \int \hat{v}_l{}'\hat{w}_j{}'\,dx = 2ki \cdot \delta_{lj}.$$

Thus, each test function directly produces a correction via its corresponding ansatz function. The test function integral is a phase correct summation of all residuals inside its support. Scaled with $(L_{jj})^{-1}$, it produces the correct amplitude for the correction. This correction is applied via the corresponding ansatz function which can be interpreted as being a ray function of the correct wave length and anplitude, starting at the index point and running in positive direction.



- -: test functions $v_l = \hat{v}_l \cdot e^{-ik(x-x_l)}$ - -: ansatz functions $w_j = \hat{w}_j \cdot e^{ik(x-x_j)}$

—: envelope functions $\hat{v}_l$ —: envelope functions $\hat{w}_j$

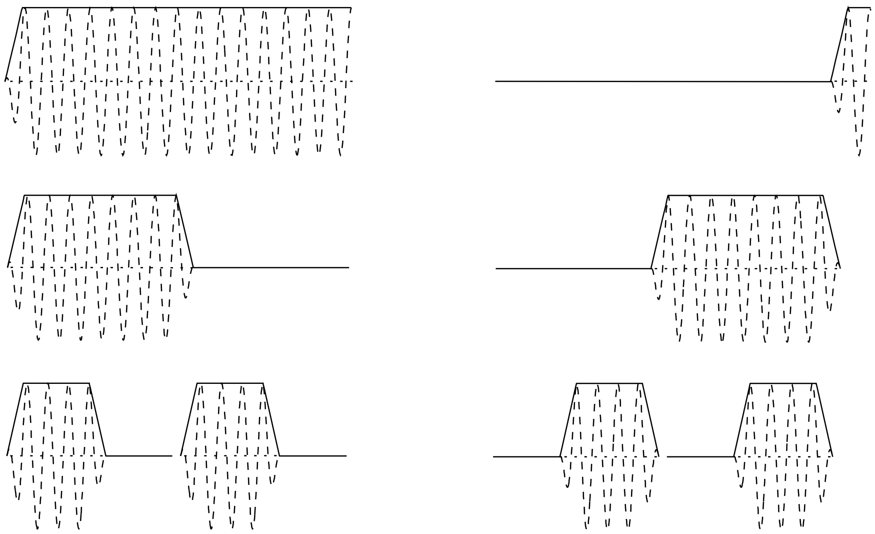
Figure 2: FEM basis functions (only the real parts are plotted) using the canonical hierarchical basis for the $\hat{w}_j$. Note that the v_l change their phase at the jump of the according envelope function $\hat{v}_l$.

5 Boundary conditions

As already mentioned, we want to use radiation boundary conditions, which in 1D mean that at the boundaries incoming waves of a fixed amplitude may be prescribed, f.e. $A \cdot e^{ikx}$ at the left or $B \cdot e^{-ikx}$ at the right boundary. The value of the outgoing waves must not be fixed at the boundary.

Therefore, when constructing the correction space we leave out the ansatz function and its according test function at the incoming boundary. See the two plots at the top of figure 4 as an example.

As the incoming waves are not touched by the correction cycles, we can set the boundary conditions by prescribing them in the initial approximation. As an alternative, one can enlarge the computational domain and reserve a few extra discretization points at the boundaries to prescribe the incoming waves.



- -: test functions $v_l = \hat{v}_l \cdot e^{-ik(x-x_l)}$
—: envelope functions $\hat{v}_l$

- -: ansatz functions $w_j = \hat{w}_j \cdot e^{ik(x-x_j)}$
—: envelope functions $\hat{w}_j$

Figure 3: The hierarchical basis based on piecewise constant functions.

6 Algorithm

Now that all the components have been introduced, we can have a look at the final algorithm and discuss some details of the implementation.

The basic algorithm is:

1. Set the initial solution (incl. the boundary conditions)
2. Compute the residual r from the finite difference equation (2)
3. For both wave directions:
   (a) Compute right hand side: $\int v_l \left(\sum r_j w_j\right) dx$
   (b) "Solve" diagonal system of equations
   (c) add the correction
4. Repeat with step 2 until the residuals are small enough

The steps (3a) and (3c) deserve special attention as we have to take some extra effort when dealing with the hierarchical bases.

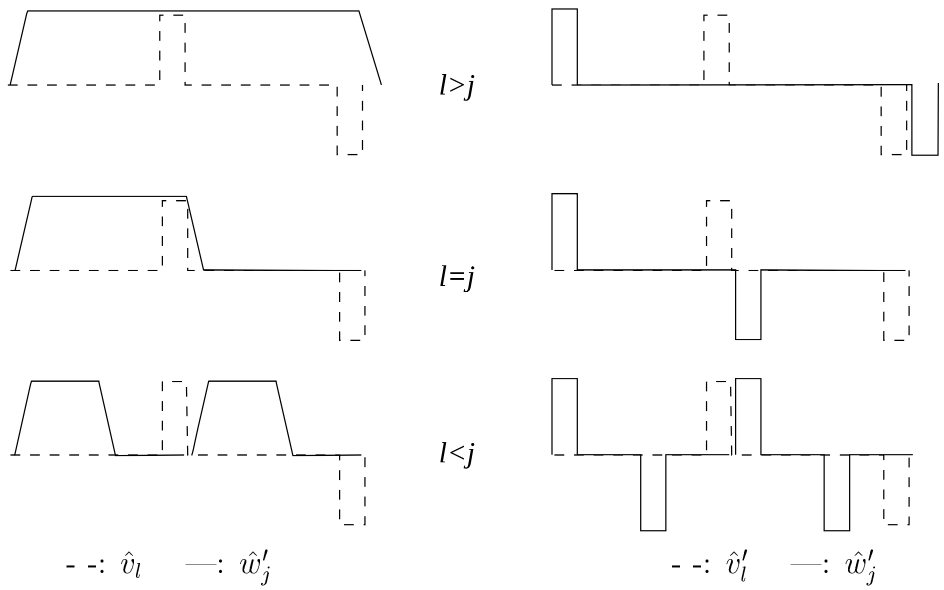


Figure 4: The second hierarchical basis.
Again the integral $\int \hat{v}_l{}'\hat{w}_j{}'\,dx$ is always zero. The integral $\int \hat{v}_l\hat{w}_j{}'\,dx$ is non-zero only for $l = j$.

The computation of the right-hand side is done via the corresponding generating system of the test space. When using the test functions from figure 2, every coarse level test functions can be computed from three lower level functions. For the e^{ikx}-correction, this is done via the following identity:

$$v_i^l(x) = e^{ik(2^{l-1}h)} v_{i-2^{l-1}}^{l-1}(x) + 2v_i^{l-1}(x) + e^{-ik(2^{l-1}h)} v_{i+2^{l-1}}^{l-1}(x). \qquad (7)$$

When using the test functions from figure 3, every test function is a summation of two lower level functions:

$$v_i^l(x) = e^{ik(2^{l-1}h)} v_{i-2^{l-1}}^{l-1}(x) + v_i^{l-1}(x). \qquad (8)$$

For the e^{-ikx}-correction, we get basically the same equations up to some sign changes.

Thus, for both bases the right-hand side integrals are first computed on the finest level (i.e. a transformed nodal base of test functions). Then the right-hand side is constructed level by level from equation (7) or (8), respectively.

After computing the ray-correction in step (3b), the correction has to be transformed back to the nodal basis. This is done in a similar way levelwise via the generating system for the ansatz space.

7 Numerical Examples

For the numerical examples, we chose the interval $[0,1]$ as the computational domain. The discretization was done with different mesh sizes $h = 2^{-\nu}, \nu = 7, 8, 9, 10$, and we took the values $k = 60, 90, 120, 150$ for the wave number.

We ran computations for two different test problems:

a) We gave the solution $u(x) = \sin^2(\pi x)e^{ikx}$ and set the incoming waves to zero as boundary condition. Then the right-hand side computes to $f(x) = \left(2\pi^2 \cos(2\pi x) + 2\pi ik \sin(2\pi x)\right) e^{ikx}$. With the initial solution $u \equiv 0$ we try to recover the given solution.

b) We set e^{ikx} as the boundary condition at the left border ($x = 0$). The right-hand side is

$$f(x) = \begin{cases} 0 & \text{if} \quad |x - 0.5| > 0.4, \\ \text{random}(x) & \text{if} \quad |x - 0.5| \leq 0.4, \end{cases}$$

where random(x) returns a random complex number with an absolute value less than k^2.

The following tables show the number of iterations necesseary to reduce the Euclidean norm of the residual by the factor 10^{-8}.

test problem a				
$k\backslash h$	2^{-7}	2^{-8}	2^{-9}	2^{-10}
60	6	4	4	3
90	8	5	4	3
120	10	6	4	4
150	13	7	5	4

test problem b				
$k\backslash h$	2^{-7}	2^{-8}	2^{-9}	2^{-10}
60	6	4	4	3
90	8	5	4	3
120	10	6	4	4
150	12	7	5	4

The next tables show the number of iterations when the product kh is constant while the mesh size is reduced:

test problem a				
$kh\backslash h$	2^{-7}	2^{-8}	2^{-9}	2^{-10}
0.125	4	4	4	4
0.25	5	5	5	5
0.5	6	6	6	6
1	11	11	11	11

test problem b				
$kh\backslash h$	2^{-7}	2^{-8}	2^{-9}	2^{-10}
0.125	4	4	4	4
0.25	5	5	5	5
0.5	6	6	6	6
1	11	11	11	11

The numbers show clearly that the convergence rate is only depending on the product kh and not on the mesh size h itself. The kh-dependency is a result of the transformation of the residual — remember the term $\sum r_j w_j(x)$ in equation (5). The convergence slows down quickly when the product kh gets larger than 1. This is no serious problem as this violates the condition that h should be small against the wavelength. For $kh > 1$, the discretized equation (2) no longer corresponds to the analytic equation (1).
For the interesting case $kh < 1$, our algorithm solves the Helmholtz problem in just a few iterations.

8 Conclusion and Outlook to a 2D scheme

In this paper we demonstrated how the ray-correction idea of Brandt and Livshits can be formulated using a specially designed hierarchical basis. For the 1D case the hierarchical ray-corrections turned out to be highly effective for removing all error components including the principal error.

Future work will see the generalization to the 2D case. We expect that the hierarchical bases provide us with a straightforward approach for the design of efficient ray-correction cycles that get by without some of the rather technical aspects of the algorithm by Brandt and Livshits. Hierarchical bases for a 2D ray-correction will focus on the principal error component in one direction each and will be the tensor product of a hierarchical basis in wave direction — like those that were introduced in this paper — and a standard nodal basis in the perpendicular direction. Since, in 2D, one cannot hope that the ray-corrections are sufficient to remove all error components, the ray-corrections will have to be added to a standard multigrid algorithm.

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